The Vector Meson And Heavy Meson Strong Interaction

Peng-Zhi Huang,* Liang Zhang,† and Shi-Lin Zhu‡

Department of Physics and State Key Laboratory of Nuclear Physics and Technology
Peking University, Beijing 100871, China

We calculate the coupling constants between the light vector mesons and heavy mesons within the framework of the light-cone QCD sum rule in the leading order of heavy quark effective theory. The sum rules are very stable with the variations of the Borel parameter and the continuum threshold. The extracted couplings will be useful in the study of the possible heavy meson molecular states. They may also helpful in the interpretation of the proximity of X(3872), Y(4260) and Z(4430) to the threshold of two charmed mesons through the couple-channel mechanism.

I. INTRODUCTION

A number of hadronic states which can not be easily accommodated in the conventional quark model have been observed experimentally in recent years, such as $X(3872)$ [1], Y(4260) [2] and $Z^+(4430)$ [3]. Their masses are very close to the thresholds of $D\bar{D}^*$, $D^*\bar{D}^*$ and $D^*\bar{D}_1$ respectively. It was speculated that the coupled-channel effect may play an important role because of the attraction between these D mesons. Alternatively, they were considered to be possible candidates of the heavy molecular states composed of two D mesons. These loosely bound states are formed by exchanging light mesons such as π, σ, ρ and ω etc. Up to now, the pion heavy meson strong interaction is relatively known due to chiral symmetry. However, the vector meson heavy meson strong interaction has not be extensively studied yet, which accounts for the relatively short distance interaction between two heavy mesons.

Heavy quark effective theory (HQET) [4] is a systematic approach to study the spectra and transition amplitudes of heavy hadrons. In HQET, the expansion is performed in term of $1/m_Q$, where m_Q is the mass of the heavy quark involved. In the limit $m_Q \to \infty$, heavy hadrons form a series of degenerate doublets due to heavy quark symmetry. The two states in a doublet share the same quantum number j_l, the angular momentum of the light components. The $B(D)$ meson doublets $(0^-, 1^-)$, $(0^+, 1^+)$ and $(1^+, 2^+)$ are conventionally denoted as H, S and T.

Light-cone QCD sum rules (LCQSR) [5] is a very useful non-perturbative approach to determine various hadronic transition form factors. One considers the T-product of the two interpolating currents sandwiched between the vacuum and an hadronic state in this framework. Now the operator product expansion (OPE) is performed near the light-cone rather than at small distance as in the conventional SVZ sum rules [6]. The double Borel transformation is always invoked to suppress the excited state and the continuum contribution.

The ρ coupling constant between D and D^* was calculated with LCQSR in full QCD in Ref. [7]. The couplings $g_{H^*H^*\rho}$, $f_{H^*H^*\rho}$, $g_{HH\rho}$ and $f_{H^*H\rho}$ were calculated in full QCD in Ref. [8]. Their values in the limit $m_Q \to \infty$ are also discussed in this paper. The ρ coupling between doublets T and H are studied in the leading order of HQET in Ref. [9].

In this work we use LCQSR to calculate the ρ coupling constants between three doublets H, S, T and within the two doublets H, S. Due to the covariant derivative in the interpolating currents of T doublet, the contribution from the 3-particle light-cone distribution amplitudes of the ρ meson has to be included when dealing with the ρ decay between doublets T and $H(S)$. We work in HQET to differentiate the two states with the same J^P value yet quite different decay widths. The interpolating currents $J_{j,P,j_l}^{\alpha_1\cdots\alpha_j}$ adopted in our work have been properly constructed in Ref. [10]. They satisfy

$$\langle 0|J_{j,P,j_l}^{\alpha_1\cdots\alpha_j}(0)|j',P',j_l'\rangle = f_{Pj_l}\delta_{jj'}\delta_{PP'}\delta_{j_lj_l'}\eta^{\alpha_1\cdots\alpha_j}, \tag{1}$$

$$i\langle 0|T\{J_{j,P,j_l}^{\alpha_1\cdots\alpha_j}(x)J_{j,P,j_l}^{\dagger\beta_1\cdots\beta_{j'}}(0)\}|0\rangle = \delta_{jj'}\delta_{PP'}\delta_{j_lj_l'}(-1)^j\mathcal{S}g_t^{\alpha_1\beta_1}\cdots g_t^{\alpha_j\beta_j}\int dt\delta(x-vt)\Pi_{P,j_l}(x), \tag{2}$$

*Electronic address: pzhuang@pku.edu.cn
†Electronic address: liangzhang@pku.edu.cn
‡Electronic address: zhusl@pku.edu.cn

in the limit $m_Q \to \infty$. Here $\eta^{\alpha_1 \cdots \alpha_j}$ is the polarization tensor for the spin j state, v is the velocity of the heavy quark, $g_t^{\alpha\beta} = g^{\alpha\beta} - v^\alpha v^\beta$, $\mathcal{S}$ denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1 \cdots \alpha_j)$ and $(\beta_1 \cdots \beta_j)$.

II.   SUM RULES FOR THE ρ COUPLING CONSTANTS

We shall perform the calculation to the leading order of HQET. According to Ref. [10], the interpolating currents for doublets H, S and T read as

$$J^\dagger_{0,-,\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_5 q, \tag{3}$$

$$J^{\dagger\alpha}_{1,-,\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_t^\alpha q, \tag{4}$$

$$J^\dagger_{0,+,\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v q, \tag{5}$$

$$J^{\dagger\alpha}_{1,+,\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_5 \gamma_t^\alpha q, \tag{6}$$

$$J^{\dagger\alpha}_{1,+,\frac{3}{2}} = \sqrt{\frac{3}{4}} \bar{h}_v \gamma_5 (-i) \left\{ \mathcal{D}_t^\alpha - \frac{1}{3} \gamma_t^\alpha \hat{\mathcal{D}}_t \right\} q, \tag{7}$$

$$J^{\dagger\alpha_1\alpha_2}_{2,+,\frac{3}{2}} = \sqrt{\frac{1}{8}} \bar{h}_v (-i) \left\{ \gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} + \gamma_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} - \frac{2}{3} g_t^{\alpha_1\alpha_2} \hat{\mathcal{D}}_t \right\} q, \tag{8}$$

where h_v is the heavy quark field in HQET, $\gamma_t^\mu \equiv \gamma^\mu - \hat{v}v^\mu$, $\mathcal{D}_t^\mu \equiv \mathcal{D}^\mu - (\mathcal{D} \cdot v)v^\mu$, $g_t^{\mu\nu} \equiv g^{\mu\nu} - v^\mu v^\nu$, and v^μ is the velocity of the heavy quark.

We consider the ρ decay of T_1 to H_1 to illustrate our calculation. Here the subscript of $T(H)$ indicates the spin of the meson involved. Owing to the conservation of the angular momentum of light components in the limit $m_Q \to \infty$, there are three independent ρ coupling constants between doublets T and H. We denote them as $g^{s1}_{TH\rho}$, $g^{d1}_{TH\rho}$ and $g^{d2}_{TH\rho}$ where $s, p, d \cdots$ and the number following them indicates the orbital and total angular momentum (l, j_h) of the final ρ meson respectively. All of these three coupling constants appear in the decay process under consideration. The decay amplitude can now be written as

$$\mathcal{M}(T_1 \to H_1 + \rho) = Ii \Bigg\{ \epsilon^{\eta\epsilon^* e^* v} g^{s1}_{T_1 H_1 \rho} + \left[\epsilon^{\eta\epsilon^* qv} (e^* \cdot q_t) - \frac{1}{3} \epsilon^{\eta\epsilon^* e^* v} q_t^2 \right] g^{d1}_{T_1 H_1 \rho}$$
$$+ \left[\epsilon^{\eta e^* qv} (\epsilon^* \cdot q_t) + \epsilon^{\epsilon^* e^* qv} (\eta \cdot q_t) \right] g^{d2}_{T_1 H_1 \rho} \Bigg\}, \tag{9}$$

where η, ϵ^* and e^* denote the polarization vector of T_1, H_1 and ρ respectively, q is the momentum of the ρ meson, $q^2 = m_\rho^2$ and $q_t^\mu \equiv q^\mu - (q \cdot v)v^\mu$. $I = 1, 1/\sqrt{2}$ for the charged and neutral ρ meson respectively. The vector notations in Levi-Civita tensor come from an index contraction between Levi-Civita tensor and the vectors, for example, $\epsilon^{\eta\epsilon^* e^* v} \equiv \epsilon^{\mu\nu\rho\sigma} \eta_\mu \epsilon_\nu^* e_\rho^* v_\sigma$.

To obtain the sum rules for the coupling constants $g^{s1}_{T_1 H_1 \rho}$, $g^{d1}_{T_1 H_1 \rho}$ and $g^{d2}_{T_1 H_1 \rho}$, we consider the correlation functions

$$\int d^4x e^{-ik \cdot x} \langle \rho(q)| T\{J^\beta_{1,-,\frac{1}{2}}(0) J^{\dagger\alpha}_{1,+,\frac{3}{2}}(x)\}|0\rangle = Ii \Bigg\{ \epsilon^{\alpha\beta e^* v} G^{s1}_{T_1 H_1 \rho}(\omega, \omega')$$
$$+ \left[\epsilon^{\alpha\beta qv} (e^* \cdot q_t) - \frac{1}{3} \epsilon^{\alpha\beta e^* v} q_t^2 \right] G^{d1}_{T_1 H_1 \rho}(\omega, \omega')$$
$$+ \left[\epsilon^{\alpha e^* qv} q_t^\beta + \epsilon^{\beta e^* qv} q_t^\alpha \right] G^{d2}_{T_1 H_1 \rho}(\omega, \omega') \Bigg\}, \tag{10}$$

where $\omega \equiv 2v \cdot k$, $\omega' \equiv 2v \cdot (k - q)$. In the leading order of HQET, the heavy quark propagator reads as

$$\langle 0| T\{h_v(0)\bar{h}_v(x)\}|0\rangle = \frac{1+\hat{v}}{2} \int dt \delta^4(-x - vt). \tag{11}$$

The correlation function can now be expressed as

$$-\sqrt{\frac{3}{8}}\int dx e^{-ik\cdot x}\int_0^\infty dt\delta(-x-vt)\text{Tr}\left\{\gamma_t^\beta\frac{1+\hat{v}}{2}(-i\gamma_5)(\mathcal{D}_t^\alpha-\frac{1}{3}\gamma^\alpha\hat{\mathcal{D}}_t)\langle\rho(q)|q(x)\bar{q}(0)|0\rangle\right\}. \tag{12}$$

It can be further calculated using the light cone wave functions of the ρ meson. To our approximation, we need the two and three-particle light-cone wave functions. Their definitions are collected in the Appendix B.

At the hadron level, the G's in (10) has the following pole terms

$$G_{T_1H_1\rho}(\omega,\omega')=\frac{f_{-,1/2}f_{+,3/2}g_{T_1H_1\rho}}{(2\bar{\Lambda}_{-,1/2}-\omega')(2\bar{\Lambda}_{+,3/2}-\omega)}+\frac{c}{2\bar{\Lambda}_{-,1/2}-\omega'}+\frac{c'}{2\bar{\Lambda}_{+,3/2}-\omega}, \tag{13}$$

where $\bar{\Lambda}_{-,1/2}\equiv m_H-m_Q$, $\bar{\Lambda}_{+,3/2}\equiv m_T-m_Q$. $f_{-,1/2}$ etc is the overlap amplitudes of their interpolating currents with the heavy mesons.

$G_{T_1H_1\rho}(\omega,\omega')$ can now be expressed by the ρ meson light-cone wave functions. After the Wick rotation and the double Borel transformation with ω and ω', the single-pole terms in (13) are eliminated. We arrive at

$$\sqrt{3}g_{T_1H_1\rho}^{s1}f_{-,\frac{1}{2}}f_{+,\frac{3}{2}}e^{-\frac{\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{-,1/2}}{T}}$$

$$\begin{aligned}
=&\ \frac{1}{3}f_\rho^T m_\rho^4 h_\parallel^{s[1]}(\bar{u}_0)\frac{1}{T}-\frac{1}{3}f_\rho^T m_\rho^4(uh_\parallel^s)^{[1]}(\bar{u}_0)\frac{1}{T}-\frac{2}{3}f_\rho^T m_\rho^4\mathcal{S}^{[-1,0]}(u_0)\frac{1}{T}+\frac{1}{24}f_\rho^T m_\rho^4 A_T(\bar{u}_0)\frac{1}{T}\\
&-\frac{1}{24}f_\rho^T m_\rho^4 A_T(\bar{u}_0)\bar{u}_0\frac{1}{T}+\frac{2}{3}f_\rho^T m_\rho^4 B_T^{[3]}(\bar{u}_0)\frac{1}{T}-\frac{1}{4}f_\rho^T m_\rho^2 C_T^{[1]}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})-\frac{1}{4}f_\rho^T m_\rho^2 h_\parallel^s(\bar{u}_0)Tf_0(\frac{\omega_c}{T})\\
&-\frac{1}{12}f_\rho^T m_\rho^2 h_\parallel^{s(1)}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{12}f_\rho^T m_\rho^2(uh_\parallel^s)'(\bar{u}_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{3}f_\rho^T m_\rho^2\mathcal{S}^{[1,0]}(u_0)Tf_0(\frac{\omega_c}{T})\\
&-\frac{1}{6}f_\rho^T m_\rho^2\varphi_\perp(\bar{u}_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{6}f_\rho^T m_\rho^2\varphi_\perp(\bar{u}_0)\bar{u}_0Tf_0(\frac{\omega_c}{T})-\frac{1}{96}f_\rho^T m_\rho^2 A_T^{(2)}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})\\
&+\frac{1}{96}f_\rho^T m_\rho^2(uA_T)^{(2)}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})-\frac{1}{6}f_\rho^T m_\rho^2 B_T^{[1]}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{24}f_\rho^T\varphi_\perp^{(2)}(\bar{u}_0)T^3f_2(\frac{\omega_c}{T})\\
&-\frac{1}{24}f_\rho^T(\varphi_\perp u)^{(2)}(\bar{u}_0)T^3f_2(\frac{\omega_c}{T})+\frac{1}{3}f_\rho^T m_\rho^4\mathcal{T}^{[-1,0]}(u_0)\frac{1}{T}-\frac{4}{3}f_\rho^T m_\rho^4\mathcal{T}_2^{[-1,0]}(u_0)\frac{1}{T}\\
&+\frac{1}{12}f_\rho^T m_\rho^2\mathcal{T}^{[1,0]}(u_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{6}f_\rho^T m_\rho^2\mathcal{T}_2^{[1,0]}(u_0)Tf_0(\frac{\omega_c}{T})-\frac{2}{3}f_\rho^T m_\rho^4\mathcal{T}_3^{[-1,0]}(u_0)\frac{1}{T}\\
&-\frac{1}{6}f_\rho^T m_\rho^2\mathcal{T}_3^{[1,0]}(u_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{12}f_\rho m_\rho^5 A^{[2]}(\bar{u}_0)\frac{1}{T^2}-\frac{1}{12}f_\rho m_\rho^5 A^{[1]}(\bar{u}_0)\frac{1}{T^2}+\frac{1}{12}f_\rho m_\rho^5(uA)^{[1]}(\bar{u}_0)\frac{1}{T^2}\\
&+\frac{4}{3}f_\rho m_\rho^5 C^{[4]}(\bar{u}_0)\frac{1}{T^2}-\frac{2}{3}f_\rho m_\rho^5 C^{[3]}(\bar{u}_0)\frac{1}{T^2}+\frac{2}{3}f_\rho m_\rho^5(uC)^{[3]}(\bar{u}_0)\frac{1}{T^2}+\frac{1}{24}f_\rho m_\rho^3 A(\bar{u}_0)\\
&+\frac{1}{48}f_\rho m_\rho^3 A'(\bar{u}_0)-\frac{1}{48}f_\rho m_\rho^3(uA)'(\bar{u}_0)-\frac{1}{3}f_\rho m_\rho^3\mathcal{A}^{[0,0]}(u_0)+\frac{1}{6}f_\rho m_\rho^3 C^{[2]}(\bar{u}_0)+\frac{1}{6}f_\rho m_\rho^3 C^{[1]}(\bar{u}_0)\\
&-\frac{1}{6}f_\rho m_\rho^3(uC)^{[1]}(\bar{u}_0)+\frac{1}{12}f_\rho m_\rho^3 g_\perp^{(a)[1]}(\bar{u}_0)-\frac{1}{12}f_\rho m_\rho^3 g_\perp^{(a)}(\bar{u}_0)+\frac{1}{12}f_\rho m_\rho^3 g_\perp^{(a)}(\bar{u}_0)\bar{u}_0\\
&+\frac{1}{3}f_\rho m_\rho^3 g_\perp^{(v)[2]}(\bar{u}_0)-\frac{1}{12}f_\rho m_\rho\mathcal{A}^{[2,0]}(u_0)T^2f_1(\frac{\omega_c}{T})+\frac{1}{2}f_\rho m_\rho^3\mathcal{V}^{[0,0]}(u_0)+\frac{1}{6}f_\rho m_\rho\mathcal{V}^{[2,0]}(u_0)T^2f_1(\frac{\omega_c}{T})\\
&-\frac{1}{3}f_\rho m_\rho^3\varphi_\parallel^{[2]}(\bar{u}_0)+\frac{1}{3}f_\rho m_\rho^3\varphi_\parallel^{[1]}(\bar{u}_0)-\frac{1}{3}f_\rho m_\rho^3(u\varphi_\parallel)^{[1]}(\bar{u}_0)+\frac{4}{3}f_\rho m_\rho^5\Psi^{[-2,0]}(u_0)\frac{1}{T^2}+\frac{1}{6}f_\rho m_\rho^3\mathcal{V}^{[0,0]}(u_0)\\
&-\frac{1}{3}f_\rho m_\rho^3\Psi^{[0,0]}(u_0)+\frac{4}{3}f_\rho m_\rho^5\tilde{\Phi}^{[-2,0]}(u_0)\frac{1}{T^2}-\frac{1}{3}f_\rho m_\rho^3\tilde{\Phi}^{[0,0]}(u_0)+\frac{1}{24}f_\rho m_\rho g_\perp^{(a)(1)}(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})\\
&+\frac{1}{48}f_\rho m_\rho g_\perp^{(a)(2)}(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})-\frac{1}{48}f_\rho m_\rho(g_\perp^{(a)}u)^{(2)}(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})+\frac{1}{6}f_\rho m_\rho g_\perp^{(v)}(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})\\
&-\frac{1}{6}f_\rho m_\rho\varphi_\parallel(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})-\frac{1}{12}f_\rho m_\rho\varphi_\parallel'(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})+\frac{1}{12}f_\rho m_\rho(u\varphi_\parallel)'(\bar{u}_0)T^2f_1(\frac{\omega_c}{T}),
\end{aligned} \tag{14}$$

$$\sqrt{3}g_{T_1H_1\rho}^{d1}f_{-,\frac{1}{2}}f_{+,\frac{3}{2}}e^{-\frac{\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{-,1/2}}{T}}$$

$$\begin{aligned}
=&\ -f_\rho^T m_\rho^2 h_\parallel^{s[1]}(\bar{u}_0)\frac{1}{T}+f_\rho^T m_\rho^2(uh_\parallel^s)^{[1]}(\bar{u}_0)\frac{1}{T}+2f_\rho^T m_\rho^2\mathcal{S}^{[-1,0]}(u_0)\frac{1}{T}+\frac{1}{16}f_\rho^T m_\rho^2 A_T(\bar{u}_0)\frac{1}{T}\\
&-\frac{1}{16}f_\rho^T m_\rho^2 A_T(\bar{u}_0)\bar{u}_0\frac{1}{T}-2f_\rho^T m_\rho^2 B_T^{[3]}(\bar{u}_0)\frac{1}{T}-\frac{1}{4}f_\rho^T\varphi_\perp(\bar{u}_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{4}f_\rho^T\varphi_\perp(\bar{u}_0)\bar{u}_0Tf_0(\frac{\omega_c}{T})
\end{aligned}$$

$$-f_\rho^T m_\rho^2 \mathcal{T}^{[-1,0]}(u_0)\frac{1}{T} + f_\rho^T m_\rho^2 \mathcal{T}_2^{[-1,0]}(u_0)\frac{1}{T} - f_\rho^T m_\rho^2 \mathcal{T}_3^{[-1,0]}(u_0)\frac{1}{T} - \frac{1}{4}f_\rho m_\rho^3 A^{[2]}(\bar{u}_0)\frac{1}{T^2}$$

$$+\frac{1}{4}f_\rho m_\rho^3 A^{[1]}(\bar{u}_0)\frac{1}{T^2} - \frac{1}{4}f_\rho m_\rho^3 (uA)^{[1]}(\bar{u}_0)\frac{1}{T^2} - 4f_\rho m_\rho^3 C^{[4]}(\bar{u}_0)\frac{1}{T^2} + 2f_\rho m_\rho^3 C^{[3]}(\bar{u}_0)\frac{1}{T^2}$$

$$-2f_\rho m_\rho^3 (uC)^{[3]}(\bar{u}_0)\frac{1}{T^2} - \frac{1}{2}f_\rho m_\rho \mathcal{A}^{[0,0]}(u_0) - \frac{1}{4}f_\rho m_\rho g_\perp^{(a)[1]}(\bar{u}_0) - \frac{1}{8}f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0)$$

$$+\frac{1}{8}f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0)\bar{u}_0 - f_\rho m_\rho g_\perp^{(v)[2]}(\bar{u}_0) + f_\rho m_\rho \mathcal{V}^{[0,0]}(u_0) + f_\rho m_\rho \varphi_\parallel^{[2]}(\bar{u}_0) - f_\rho m_\rho \varphi_\parallel^{[1]}(\bar{u}_0)$$

$$+f_\rho m_\rho (u\varphi_\parallel)^{[1]}(\bar{u}_0) - 4f_\rho m_\rho^3 \Psi^{[-2,0]}(u_0)\frac{1}{T^2} - 4f_\rho m_\rho^3 \tilde{\Phi}^{[-2,0]}(u_0)\frac{1}{T^2}, \tag{15}$$

$$\sqrt{3}g_{T_1 H_1 \rho}^{d2} f_{-,\frac{1}{2}} f_{+,\frac{3}{2}} e^{-\frac{\bar{\Lambda}_{+,3/2} + \bar{\Lambda}_{-,1/2}}{T}}$$

$$= \frac{3}{16}f_\rho^T m_\rho^2 A_T(\bar{u}_0)\frac{1}{T} - \frac{3}{16}f_\rho^T m_\rho^2 A_T(\bar{u}_0)\bar{u}_0\frac{1}{T} - \frac{3}{4}f_\rho^T \varphi_\perp(\bar{u}_0)T f_0(\frac{\omega_c}{T}) + \frac{3}{4}f_\rho^T \varphi_\perp(\bar{u}_0)\bar{u}_0 T f_0(\frac{\omega_c}{T})$$

$$-3f_\rho^T m_\rho^2 \mathcal{T}_2^{[-1,0]}(u_0)\frac{1}{T} - 3f_\rho^T m_\rho^2 \mathcal{T}_3^{[-1,0]}(u_0)\frac{1}{T} + \frac{3}{2}f_\rho m_\rho \mathcal{A}^{[0,0]}(u_0) - \frac{3}{8}f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0) + \frac{3}{8}f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0)\bar{u}_0, \tag{16}$$

where $f_n(x) = 1 - e^{-x}\sum_{k=0}^n \frac{x^k}{k!}$ is the continuum subtraction factor, and ω_c is the continuum threshold, $u_0 = \frac{T_1}{T_1+T_2}$, $T = \frac{T_1 T_2}{T_1+T_2}$ and $\bar{u}_0 = 1 - u_0$. T_1 and T_2 are the two Borel parameters. We have used the Borel transformation $\tilde{\mathcal{B}}_\omega^T e^{\alpha\omega} = \delta(\alpha - \frac{1}{T})$ to obtain (14),(15) and (16). In the above expressions we have used the following functions $\mathcal{F}^{[a]}(\bar{u}_0)$ and $\mathcal{F}^{[a,b]}(u_0)$. They are defined as

$$\mathcal{F}^{[n]}(\bar{u}_0) \equiv \int_0^{\bar{u}_0} \cdots \int_0^{x_3} \int_0^{x_2} \mathcal{F}(x_1) dx_1 dx_2 \cdots dx_n, \tag{17}$$

$$\mathcal{F}^{[0,0]}(u_0) \equiv \int_0^{u_0} \int_{u_0-\alpha_2}^{1-\alpha_2} \frac{\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)}{\alpha_3} d\alpha_3 d\alpha_2, \tag{18}$$

$$\mathcal{F}^{[1,0]}(u_0) \equiv \int_0^{u_0} \frac{\mathcal{F}(1-u_0,\alpha_2,u_0-\alpha_2)}{u_0-\alpha_2} d\alpha_2 - \int_0^{1-u_0} \frac{\mathcal{F}(u_0,1-u_0-\alpha_3,\alpha_3)}{\alpha_3} d\alpha_3, \tag{19}$$

$$\mathcal{F}^{[2,0]}(u_0) \equiv \int_0^{u_0} d\alpha_2 \frac{\partial[\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)]/\partial\alpha_2}{\alpha_3}\Big|_{\alpha_3=u_0-\alpha_2}$$

$$-\int_0^{1-u_0} d\alpha_3 \frac{\partial[\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)]/\partial\alpha_2}{\alpha_3}\Big|_{\alpha_2=u_0}, \tag{20}$$

$$\mathcal{F}^{[-1,0]}(u_0) \equiv \int_0^{u_0} \int_0^{u_0-\alpha_2} \mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3) d\alpha_3 d\alpha_2$$

$$+\int_0^{u_0} \int_{u_0-\alpha_2}^{1-\alpha_2} \frac{(u_0-\alpha_2)\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)}{\alpha_3} d\alpha_3 d\alpha_2, \tag{21}$$

$$\mathcal{F}^{[-2,0]}(u_0) \equiv \int_0^{u_0} \int_0^{u_0-\alpha_2} \int_0^{\alpha_3} \mathcal{F}(1-\alpha_2-x,\alpha_2,x) dx d\alpha_3 d\alpha_2$$

$$+\frac{1}{2}\int_0^{u_0} \int_0^{u_0-\alpha_2} \alpha_3 \mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3) d\alpha_3 d\alpha_2$$

$$+\frac{1}{2}\int_0^{u_0} \int_{u_0-\alpha_2}^{1-\alpha_2} \frac{(u_0-\alpha_2)^2}{\alpha_3} \mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3) d\alpha_3 d\alpha_2, \tag{22}$$

$$\mathcal{F}^{[-3,0]}(u_0) \equiv \int_0^{u_0} \int_0^{u_0-\alpha_2} \int_0^{\alpha_3} \int_0^{x_2} \mathcal{F}(1-\alpha_2-x_1,\alpha_2,x_1) dx_1 dx_2 d\alpha_3 d\alpha_2$$

$$+\frac{1}{2}\int_0^{u_0} \int_0^{u_0-\alpha_2} \int_0^{\alpha_3} x\mathcal{F}(1-\alpha_2-x,\alpha_2,x) dx d\alpha_3 d\alpha_2$$

$$+\frac{1}{6}\int_0^{u_0} \int_0^{u_0-\alpha_2} \alpha_3^2 \mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3) d\alpha_3 d\alpha_2$$

$$+\frac{1}{6}\int_0^{u_0} \int_{u_0-\alpha_2}^{1-\alpha_2} \frac{(u_0-\alpha_2)^3}{\alpha_3} \mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3) d\alpha_3 d\alpha_2, \tag{23}$$

III. NUMERICAL ANALYSIS

In our numerical analysis, we need the mass parameters $\bar{\Lambda}$'s and f's, the overlapping amplitudes of these interpolating currents. We adopt $\bar{\Lambda}_{-,1/2}$ from Ref. [11]: $\bar{\Lambda}_{-,1/2} = 0.5$ GeV, $f_{-,1/2} = 0.25 \pm 0.04$ GeV$^{3/2}$. $\bar{\Lambda}_{+,1/2}$, $f_{+,1/2}$, $\bar{\Lambda}_{+,3/2}$, and $f_{+,3/2}$ are given in Ref. [12]:

$$\bar{\Lambda}_{+,1/2} = 1.15 \text{ GeV}, \quad f_{+,1/2} = -0.40 \pm 0.06 \text{ GeV}^{3/2},$$
$$\bar{\Lambda}_{+,3/2} = 0.82 \text{ GeV}, \quad f_{+,3/2} = 0.19 \pm 0.03 \text{ GeV}^{5/2}.$$

The parameters appear in the distribution amplitudes of the ρ meson take the values from Ref. [14]. We use the values at the scale $\mu = 1$ GeV in our calculation under the consideration that the heavy quark behaves almost as a spectator of the decay processes in our discussion in the leading order of HQET:

$f_\rho^\parallel$[MeV]	$f_\rho^\perp$[MeV]	$a_2^\parallel$	$a_2^\perp$	$\zeta_{3\rho}^\parallel$	$\tilde{\omega}_{3\rho}^\parallel$	$\omega_{3\rho}^\parallel$	$\omega_{3\rho}^\perp$	$\zeta_4^\parallel$	$\tilde{\omega}_4^\parallel$	$\zeta_4^\perp$	$\tilde{\zeta}_4^\perp$
216(3)	165(9)	0.15(7)	0.14(6)	0.030(10)	$-0.09(3)$	0.15(5)	0.55(25)	0.07(3)	$-0.03(1)$	$-0.03(5)$	$-0.08(5)$

We will work at the symmetry point, i.e., $T_1 = T_2 = 2T$, $u_0 = 1/2$. This comes from the observation that the mass differences between H, S and T are less than 0.4 GeV in the leading order of HQET. They are much smaller than the Borel parameter T_1, $T_2 \sim 3$ GeV used below. On the other hand, every reliable sum rule has a working window of the Borel parameter T within which the sum rule is insensitive to the variation of T. So it is reasonable to choose a common point $T_1 = T_2$ at the overlapping region of T_1 and T_2. Furthermore, choosing $T_1 = T_2$ will enable us to subtract the continuum contribution cleanly while the asymmetric choice will lead to the continuum substraction very difficult [15].

From the requirement that the pole contribution is larger than 60%, we get the upper bound of the Borel parameter. This leads to $T < 1.7$ GeV. The convergence requirement of the operator product expansion leads to the lower bound of the Borel parameter $T = 1.3$ GeV, starting from which the stability of the sum rule develops. The resulting sum rules are plotted in Fig.1,2 and 3 in the working interval 1.3 GeV $< T < 1.7$ GeV and $\omega_c = 2.8, 3.0, 3.2$ GeV.

1
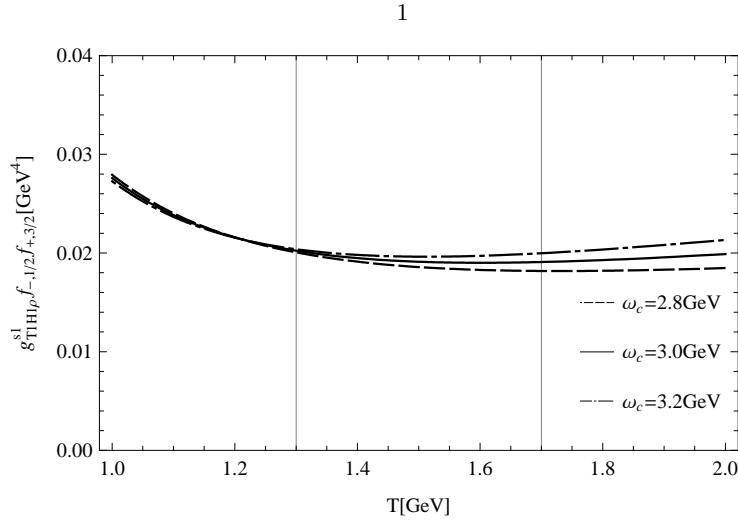

FIG. 1: The sum rule for $g_{T_1 H_1 \rho}^{s1} f_{-,1/2} f_{+,3/2}$ with $\omega_c = 2.8, 3.0, 3.2$GeV

Other ρ coupling constants between H, S and T doublets can be calculated in the same way as $g_{T_1 H_1 \rho}$. Their definitions and the relevant correlators are given in Appendix A. Here we simply present the sum rules for these coupling constants:

$$g_{H_1 H_1 \rho}^{p0} f_{-,\frac{1}{2}}^2 = \frac{1}{2\sqrt{2}} e^{\frac{2\bar{\Lambda}_{-,1/2}}{T}} \left\{ -f_\rho m_\rho^3 A^{[1]}(\bar{u}_0)\frac{1}{T^2} - 8f_\rho m_\rho^3 C^{[3]}(\bar{u}_0)\frac{1}{T^2} \right.$$
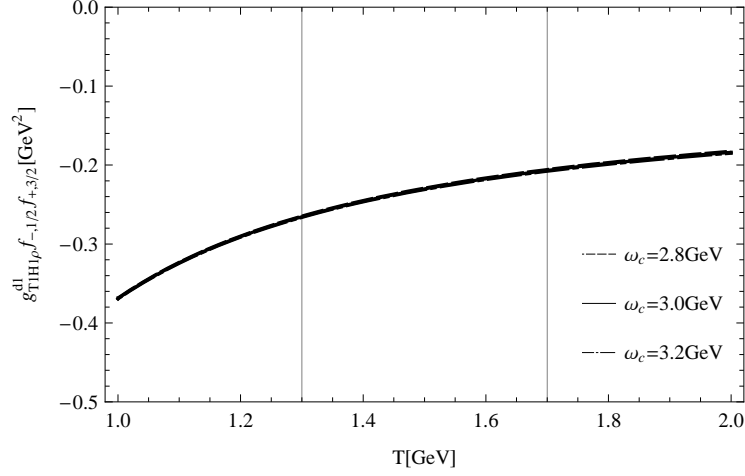
2

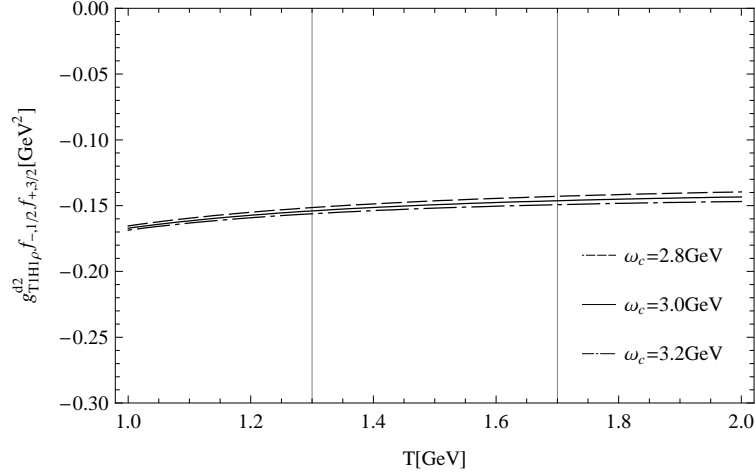FIG. 2: The sum rule for $g_{T_1H_1\rho}^{d1}f_{-,1/2}f_{+,3/2}$ with $\omega_c = 2.8, 3.0, 3.2\text{GeV}$

3

FIG. 3: The sum rule for $g_{T_1H_1\rho}^{d2}f_{-,1/2}f_{+,3/2}$ with $\omega_c = 2.8, 3.0, 3.2\text{GeV}$

$$+4f_\rho^T m_\rho^2 h_\parallel^{s[1]}(\bar{u}_0)\frac{1}{T} + 4f_\rho m_\rho \varphi_\parallel^{[1]}(\bar{u}_0)\bigg\}, \tag{24}$$

$$g_{H_1H_1\rho}^{p1}f_{-,\frac{1}{2}}^2 = \frac{1}{4\sqrt{2}}e^{\frac{2\bar{\Lambda}_{-,1/2}}{T}}\bigg\{f_\rho^T m_\rho^2 A_T(\bar{u}_0)\frac{1}{T} - 2f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0) - 4f_\rho^T \varphi_\perp(\bar{u}_0)Tf_0(\frac{\omega_c}{T})\bigg\}, \tag{25}$$

$$g_{S_1S_1\rho}^{p0}f_{+,\frac{1}{2}}^2 = \frac{1}{2\sqrt{2}}e^{\frac{2\bar{\Lambda}_{+,1/2}}{T}}\bigg\{-4f_\rho^T m_\rho^2 h_\parallel^{s[1]}(\bar{u}_0)\frac{1}{T} - f_\rho m_\rho^3 A^{[1]}(\bar{u}_0)\frac{1}{T^2}$$

$$-8f_\rho m_\rho^3 C^{[3]}(\bar{u}_0)\frac{1}{T^2} + 4f_\rho m_\rho \varphi_\parallel^{[1]}(\bar{u}_0)\bigg\}, \tag{26}$$

$$g_{S_1S_1\rho}^{p1}f_{+,\frac{1}{2}}^2 = \frac{1}{4\sqrt{2}}e^{\frac{2\bar{\Lambda}_{+,1/2}}{T}}\bigg\{-f_\rho^T m_\rho^2 A_T(\bar{u}_0)\frac{1}{T} + 4f_\rho^T \varphi_\perp(\bar{u}_0)Tf_0(\frac{\omega_c}{T}) - 2f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0)\bigg\}, \tag{27}$$

$$g_{S_0H_1\rho}^{s1}f_{-,\frac{1}{2}}f_{+,\frac{1}{2}} = \frac{1}{8\sqrt{2}}e^{\frac{\bar{\Lambda}_{+,1/2}+\bar{\Lambda}_{-,1/2}}{T}}\bigg\{-8f_\rho^T m_\rho^2 C_T^{[1]}(\bar{u}_0) + f_\rho^T m_\rho^2 A_T'(\bar{u}_0)$$

$$-4f_\rho^T\varphi_\perp'(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})-8f_\rho m_\rho g_\perp^{(v)}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})\Big\},\tag{28}$$

$$g_{S_0H_1\rho}^{d1}f_{-,\frac{1}{2}}f_{+,\frac{1}{2}} = \frac{1}{2\sqrt{2}}e^{\frac{\bar{\Lambda}_{+,1/2}+\bar{\Lambda}_{-,1/2}}{T}}\Big\{-4f_\rho^T\varphi_\perp^{[1]}(\bar{u}_0)+f_\rho^Tm_\rho^2A_T^{[1]}(\bar{u}_0)\frac{1}{T^2}+16f_\rho^Tm_\rho^2B_T^{[3]}(\bar{u}_0)\frac{1}{T^2}$$

$$-2A^{[2]}(\bar{u}_0)f_\rho m_\rho^3\frac{1}{T^3}-8f_\rho m_\rho g_\perp^{(v)[2]}(\bar{u}_0)\frac{1}{T}+8f_\rho m_\rho\varphi_\parallel^{[2]}(\bar{u}_0)\frac{1}{T}\Big\},\tag{29}$$

$$\sqrt{3}g_{T_1S_1\rho}^{p1}f_{+,\frac{1}{2}}f_{+,\frac{3}{2}}e^{-\frac{\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{+,1/2}}{T}}$$

$$= -\frac{1}{4}f_\rho^Tm_\rho^2C_T^{[2]}(\bar{u}_0)+\frac{1}{4}f_\rho^Tm_\rho^2C_T^{[1]}(\bar{u}_0)-\frac{1}{4}f_\rho^Tm_\rho^2(uC_T)^{[1]}(\bar{u}_0)-f_\rho^Tm_\rho^2\tilde{\mathcal{S}}^{[0,0]}(u_0)$$

$$-\frac{1}{32}f_\rho^Tm_\rho^2A_T'(\bar{u}_0)+\frac{1}{32}f_\rho^Tm_\rho^2(uA_T)'(\bar{u}_0)-\frac{1}{2}f_\rho^Tm_\rho^2B_T^{[2]}(\bar{u}_0)+\frac{1}{8}f_\rho^T\varphi_\perp'(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})$$

$$-\frac{1}{8}f_\rho^T(u\varphi_\perp)'(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})-\frac{1}{2}f_\rho^Tm_\rho^2\mathcal{T}_3^{[0,0]}(u_0)+\frac{1}{2}f_\rho^Tm_\rho^2\mathcal{T}_4^{[0,0]}(u_0)-\frac{3}{16}f_\rho m_\rho^3A^{[1]}(\bar{u}_0)\frac{1}{T}$$

$$-f_\rho m_\rho^3\mathcal{A}^{[-1,0]}(u_0)\frac{1}{T}+\frac{1}{2}f_\rho m_\rho^3\mathcal{V}^{[-1,0]}(u_0)\frac{1}{T}-\frac{1}{2}f_\rho m_\rho\mathcal{A}^{[1,0]}(u_0)Tf_0(\frac{\omega_c}{T})$$

$$-\frac{1}{8}f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})-\frac{1}{4}f_\rho m_\rho g_\perp^{(v)[1]}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})-\frac{1}{4}f_\rho m_\rho g_\perp^{(v)}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})$$

$$+\frac{1}{4}f_\rho m_\rho g_\perp^{(v)}(\bar{u}_0)\bar{u}_0Tf_0(\frac{\omega_c}{T})+\frac{1}{4}f_\rho m_\rho\mathcal{V}^{[1,0]}(u_0)Tf_0(\frac{\omega_c}{T})+\frac{1}{4}f_\rho m_\rho\varphi_\parallel^{[1]}(\bar{u}_0)Tf_0(\frac{\omega_c}{T}),\tag{30}$$

$$\sqrt{3}g_{T_1S_1\rho}^{p2}f_{+,\frac{1}{2}}f_{+,\frac{3}{2}}e^{-\frac{\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{+,1/2}}{T}}$$

$$= \frac{3}{4}f_\rho^Tm_\rho^2C_T^{[2]}(\bar{u}_0)-\frac{3}{4}f_\rho^Tm_\rho^2C_T^{[1]}(\bar{u}_0)+\frac{3}{4}f_\rho^Tm_\rho^2(uC_T)^{[1]}(\bar{u}_0)-\frac{3}{5}f_\rho^Tm_\rho^2\varphi_\perp^{[1]}(\bar{u}_0)$$

$$+\frac{3}{5}f_\rho^Tm_\rho^2(u\varphi_\perp)^{[1]}(\bar{u}_0)+\frac{3}{20}f_\rho^Tm_\rho^4A_T^{[1]}(\bar{u}_0)\frac{1}{T^2}-\frac{3}{20}f_\rho^Tm_\rho^4(uA_T)^{[1]}(\bar{u}_0)\frac{1}{T^2}+\frac{9}{160}f_\rho^Tm_\rho^2A_T'(\bar{u}_0)$$

$$-\frac{9}{160}f_\rho^Tm_\rho^2(uA_T)'(\bar{u}_0)-\frac{24}{5}f_\rho^Tm_\rho^4B_T^{[4]}(\bar{u}_0)\frac{1}{T^2}+\frac{12}{5}f_\rho^Tm_\rho^4B_T^{[3]}(\bar{u}_0)\frac{1}{T^2}$$

$$-\frac{12}{5}f_\rho^Tm_\rho^4(uB_T)^{[3]}(\bar{u}_0)\frac{1}{T^2}-\frac{3}{10}f_\rho^Tm_\rho^2B_T^{[2]}(\bar{u}_0)-\frac{3}{5}f_\rho^Tm_\rho^2B_T^{[1]}(\bar{u}_0)+\frac{3}{5}f_\rho^Tm_\rho^2(uB_T)^{[1]}(\bar{u}_0)$$

$$-\frac{9}{40}f_\rho^T\varphi_\perp'(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})+\frac{9}{40}f_\rho^T(u\varphi_\perp)'(\bar{u}_0)T^2f_1(\frac{\omega_c}{T})+\frac{6}{5}f_\rho^Tm_\rho^4\mathcal{T}^{[-2,0]}(u_0)\frac{1}{T^2}-\frac{3}{10}f_\rho^Tm_\rho^2\mathcal{T}^{[0,0]}(u_0)$$

$$+\frac{12}{5}f_\rho^Tm_\rho^4\mathcal{T}_3^{[-2,0]}(u_0)\frac{1}{T^2}+\frac{9}{10}f_\rho^Tm_\rho^2\mathcal{T}_3^{[0,0]}(u_0)+\frac{12}{5}f_\rho^Tm_\rho^4\mathcal{T}_4^{[-2,0]}(u_0)\frac{1}{T^2}+\frac{9}{10}f_\rho^Tm_\rho^2\mathcal{T}_4^{[0,0]}(u_0)$$

$$-\frac{3}{10}f_\rho m_\rho^5A^{[3]}(\bar{u}_0)\frac{1}{T^3}+\frac{3}{10}f_\rho m_\rho^5A^{[2]}(\bar{u}_0)\frac{1}{T^3}-\frac{3}{10}f_\rho m_\rho^5(uA)^{[2]}(\bar{u}_0)\frac{1}{T^3}-\frac{9}{80}f_\rho m_\rho^3A^{[1]}(\bar{u}_0)\frac{1}{T}$$

$$-\frac{3}{40}f_\rho m_\rho^3A(\bar{u}_0)\frac{1}{T}+\frac{3}{40}f_\rho m_\rho^3A(\bar{u}_0)\bar{u}_0\frac{1}{T}-\frac{6}{5}f_\rho m_\rho^3g_\perp^{(v)[3]}(\bar{u}_0)\frac{1}{T}+\frac{6}{5}f_\rho m_\rho^3g_\perp^{(v)[2]}(\bar{u}_0)\frac{1}{T}$$

$$-\frac{6}{5}f_\rho m_\rho^3(ug_\perp^{(v)})^{[2]}(\bar{u}_0)\frac{1}{T}+\frac{9}{10}f_\rho m_\rho^3\mathcal{V}^{[-1,0]}(u_0)\frac{1}{T}+\frac{6}{5}f_\rho m_\rho^3\varphi_\parallel^{[3]}(\bar{u}_0)\frac{1}{T}-\frac{6}{5}f_\rho m_\rho^3\varphi_\parallel^{[2]}(\bar{u}_0)\frac{1}{T}$$

$$+\frac{6}{5}f_\rho m_\rho^3(u\varphi_\parallel)^{[2]}(\bar{u}_0)\frac{1}{T}-\frac{12}{5}f_\rho m_\rho^5\mathcal{V}^{[-3,0]}(u_0)\frac{1}{T^3}-\frac{24}{5}f_\rho m_\rho^5\Phi^{[-3,0]}(u_0)\frac{1}{T^3}+\frac{24}{5}f_\rho m_\rho^5\Psi^{[-3,0]}(u_0)\frac{1}{T^3}$$

$$+\frac{6}{5}f_\rho m_\rho^3\Phi^{[-1,0]}(u_0)\frac{1}{T}-\frac{6}{5}f_\rho m_\rho^3\Psi^{[-1,0]}(u_0)\frac{1}{T}-\frac{9}{20}f_\rho m_\rho g_\perp^{(v)[1]}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})$$

$$+\frac{9}{20}f_\rho m_\rho g_\perp^{(v)}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})-\frac{9}{20}f_\rho m_\rho g_\perp^{(v)}(\bar{u}_0)\bar{u}_0Tf_0(\frac{\omega_c}{T})+\frac{9}{20}f_\rho m_\rho\mathcal{V}^{[1,0]}(u_0)Tf_0(\frac{\omega_c}{T})$$

$$+\frac{9}{20}f_\rho m_\rho\varphi_\parallel^{[1]}(\bar{u}_0)Tf_0(\frac{\omega_c}{T})+\frac{3}{10}f_\rho m_\rho\varphi_\parallel(\bar{u}_0)Tf_0(\frac{\omega_c}{T})-\frac{3}{10}f_\rho m_\rho\varphi_\parallel(\bar{u}_0)\bar{u}_0Tf_0(\frac{\omega_c}{T}),\tag{31}$$

$$\sqrt{3}g_{T_1S_1\rho}^{f2}f_{+,\frac{1}{2}}f_{+,\frac{3}{2}}e^{-\frac{\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{+,1/2}}{T}}$$

$$= -3f_\rho^T\varphi_\perp^{[1]}(\bar{u}_0)+3f_\rho^T(u\varphi_\perp)^{[1]}(\bar{u}_0)+\frac{3}{4}f_\rho^Tm_\rho^2A_T^{[1]}(\bar{u}_0)\frac{1}{T^2}-\frac{3}{4}m_\rho^2(uA_T)^{[1]}(\bar{u}_0)f_\rho^T\frac{1}{T^2}$$

$$-24f_\rho^T m_\rho^2 B_T^{[4]}(\bar{u}_0)\frac{1}{T^2} + 12f_\rho^T m_\rho^2 B_T^{[3]}(\bar{u}_0)\frac{1}{T^2} - 12f_\rho^T m_\rho^2 (uB_T)^{[3]}(\bar{u}_0)\frac{1}{T^2} + 6f_\rho^T m_\rho^2 \mathcal{T}^{[-2,0]}(u_0)\frac{1}{T^2}$$

$$+12f_\rho^T m_\rho^2 \mathcal{T}_3^{[-2,0]}(u_0)\frac{1}{T^2} + 12f_\rho^T m_\rho^2 \mathcal{T}_4^{[-2,0]}(u_0)\frac{1}{T^2} - \frac{3}{2}f_\rho m_\rho^3 A^{[3]}(\bar{u}_0)\frac{1}{T^3} + \frac{3}{2}f_\rho m_\rho^3 A^{[2]}(\bar{u}_0)\frac{1}{T^3}$$

$$-\frac{3}{2}f_\rho m_\rho^3 (uA)^{[2]}(\bar{u}_0)\frac{1}{T^3} - 6f_\rho m_\rho g_\perp^{(v)[3]}(\bar{u}_0)\frac{1}{T} + 6f_\rho m_\rho g_\perp^{(v)[2]}(\bar{u}_0)\frac{1}{T} - 6f_\rho m_\rho (ug_\perp^{(v)})^{[2]}(\bar{u}_0)\frac{1}{T}$$

$$-6f_\rho m_\rho \mathcal{V}^{[-1,0]}(u_0)\frac{1}{T} + 6f_\rho m_\rho \varphi_\parallel^{[3]}(\bar{u}_0)\frac{1}{T} - 6f_\rho m_\rho \varphi_\parallel^{[2]}(\bar{u}_0)\frac{1}{T} + 6f_\rho m_\rho (u\varphi_\parallel)^{[2]}(\bar{u}_0)\frac{1}{T}$$

$$-12f_\rho m_\rho^3 \mathcal{V}^{[-3,0]}(u_0)\frac{1}{T^3} - 24f_\rho m_\rho^3 \Phi^{[-3,0]}(u_0)\frac{1}{T^3} + 24f_\rho m_\rho^3 \Psi^{[-3,0]}(u_0)\frac{1}{T^3}, \tag{32}$$

Due to heavy quark symmetry, the ρ coupling constants with the same (l, j_h) between two doublets are not independent in the leading order of HQET. The values of these coupling constants multiplied by the decay constants of the initial and the final heavy mesons are:

$$\tilde{g}_{H_0 H_0 \rho}^{p_0} = -\tilde{g}_{H_1 H_1 \rho}^{p_0} = -0.32 \pm 0.04 \text{ GeV}^2,$$

$$\tilde{g}_{H_1 H_0 \rho}^{p_1} = \tilde{g}_{H_1 H_1 \rho}^{p_1} = -0.46 \pm 0.01 \text{ GeV}^2,$$

$$\tilde{g}_{S_0 H_1 \rho}^{s_1} = -\tilde{g}_{S_1 H_0 \rho}^{s_1} = -\tilde{g}_{S_1 H_1 \rho}^{s_1} = -0.39 \pm 0.03 \text{ GeV}^3,$$

$$\tilde{g}_{S_0 H_1 \rho}^{d_1} = -\tilde{g}_{S_1 H_0 \rho}^{d_1} = -\tilde{g}_{S_1 H_1 \rho}^{d_1} = -0.38 \pm 0.06 \text{ GeV},$$

$$\tilde{g}_{S_0 S_0 \rho}^{p_0} = -\tilde{g}_{S_1 S_1 \rho}^{p_0} = -0.32 \pm 0.03 \text{ GeV}^2,$$

$$\tilde{g}_{S_1 S_0 \rho}^{p_1} = -\tilde{g}_{S_1 S_1 \rho}^{p_1} = -0.45 \pm 0.02 \text{ GeV}^2,$$

$$\tilde{g}_{T_1 H_0 \rho}^{s_1} = -2\tilde{g}_{T_1 H_1 \rho}^{s_1} = -2\sqrt{\frac{2}{3}}\tilde{g}_{T_2 H_1 \rho}^{s_1} = -0.04 \pm 0.002 \text{ GeV}^4,$$

$$\tilde{g}_{T_1 H_0 \rho}^{d_1} = 2\tilde{g}_{T_1 H_1 \rho}^{d_1} = -2\sqrt{\frac{2}{3}}\tilde{g}_{T_2 H_1 \rho}^{d_1} = -0.47 \pm 0.06 \text{ GeV}^2,$$

$$\tilde{g}_{T_1 H_1 \rho}^{d_2} = \sqrt{\frac{3}{2}}\tilde{g}_{T_2 H_0 \rho}^{d_2} = \sqrt{6}\tilde{g}_{T_2 H_1 \rho}^{d_2} = -0.15 \pm 0.01 \text{ GeV}^2,$$

$$\tilde{g}_{T_1 S_0 \rho}^{p_1} = 2\tilde{g}_{T_1 S_1 \rho}^{p_1} = 2\sqrt{\frac{2}{3}}\tilde{g}_{T_2 S_1 \rho}^{p_1} = 0.27 \pm 0.02 \text{ GeV}^3,$$

$$\tilde{g}_{T_1 S_1 \rho}^{p_2} = -\sqrt{\frac{3}{2}}\tilde{g}_{T_2 S_0 \rho}^{p_2} = -\sqrt{6}\tilde{g}_{T_2 S_1 \rho}^{p_2} = -0.28 \pm 0.02 \text{ GeV}^3,$$

$$\tilde{g}_{T_1 S_1 \rho}^{f_2} = -\sqrt{\frac{3}{2}}\tilde{g}_{T_2 S_0 \rho}^{f_2} = -\sqrt{6}\tilde{g}_{T_2 S_1 \rho}^{f_2} = 0.31 \pm 0.04 \text{ GeV}, \tag{33}$$

where $\tilde{g}_{H_0 H_0}^{p_0} \equiv g_{H_0 H_0}^{p_0} f_{-,1/2}^2$ etc. The errors come from the variations of T and ω_c in the working region and the central value corresponds to $T = 1.5\text{GeV}$ and $\omega_c = 3.0\text{GeV}$. The g's with their errors are

$$g_{H_0 H_0 \rho}^{p_0} = -g_{H_1 H_1 \rho}^{p_0} = -5.1 \pm 0.6 \pm 1.3 \text{ GeV}^{-1},$$

$$g_{H_1 H_0 \rho}^{p_1} = g_{H_1 H_1 \rho}^{p_1} = -7.4 \pm 0.2 \pm 1.8 \text{ GeV}^{-1},$$

$$g_{S_0 H_1 \rho}^{s_1} = -g_{S_1 H_0 \rho}^{s_1} = -g_{S_1 H_1 \rho}^{s_1} = 3.9 \pm 0.3 \pm 1.0,$$

$$g_{S_0 H_1 \rho}^{d_1} = -g_{S_1 H_0 \rho}^{d_1} = -g_{S_1 H_1 \rho}^{d_1} = 3.8 \pm 0.6 \pm 0.9 \text{ GeV}^{-2},$$

$$g_{S_0 S_0 \rho}^{p_0} = -g_{S_1 S_1 \rho}^{p_0} = -2.0 \pm 0.2 \pm 0.5 \text{ GeV}^{-1},$$

$$g_{S_1 S_0 \rho}^{p_1} = -g_{S_1 S_1 \rho}^{p_1} = -2.8 \pm 0.2 \pm 0.7 \text{ GeV}^{-1},$$

$$g_{T_1 H_0 \rho}^{s_1} = -2g_{T_1 H_1 \rho}^{s_1} = -2\sqrt{\frac{2}{3}}g_{T_2 H_1 \rho}^{s_1} = -0.8 \pm 0.05 \pm 0.2,$$

$$g_{T_1 H_0 \rho}^{d_1} = 2g_{T_1 H_1 \rho}^{d_1} = -2\sqrt{\frac{2}{3}}g_{T_2 H_1 \rho}^{d_1} = -9.9 \pm 1.3 \pm 2.5 \text{ GeV}^{-2},$$

$$g_{T_1 H_1 \rho}^{d_2} = \sqrt{\frac{3}{2}}g_{T_2 H_0 \rho}^{d_2} = \sqrt{6}g_{T_2 H_1 \rho}^{d_2} = -3.1 \pm 0.1 \pm 0.8 \text{ GeV}^{-2},$$

$$g^{p_1}_{T_1 S_0 \rho} = 2g^{p_1}_{T_1 S_1 \rho} = 2\sqrt{\frac{2}{3}}g^{p_1}_{T_2 S_1 \rho} = 3.5 \pm 0.3 \pm 0.9 \text{ GeV}^{-1},$$

$$g^{p_2}_{T_1 S_1 \rho} = -\sqrt{\frac{3}{2}}g^{p_2}_{T_2 S_0 \rho} = -\sqrt{6}g^{p_2}_{T_2 S_1 \rho} = -3.7 \pm 0.3 \pm 0.9 \text{ GeV}^{-1},$$

$$g^{f_2}_{T_1 S_1 \rho} = -\sqrt{\frac{3}{2}}g^{f_2}_{T_2 S_0 \rho} = -\sqrt{6}g^{f_2}_{T_2 S_1 \rho} = 4.1 \pm 0.5 \pm 1.0 \text{ GeV}^{-3}. \tag{34}$$

The second error comes from the uncertainty of f's. The above relations between coupling constants are consistent with the HQET leading order expectation.

Replacing the ρ meson parameters by those for the ω meson, one obtains the ω meson couplings with the heavy mesons:

$$\tilde{g}^{p_0}_{H_0 H_0 \omega} = -\tilde{g}^{p_0}_{H_1 H_1 \omega} = -0.29 \pm 0.04 \text{ GeV}^2,$$

$$\tilde{g}^{p_1}_{H_1 H_0 \omega} = \tilde{g}^{p_1}_{H_1 H_1 \omega} = -0.41 \pm 0.01 \text{ GeV}^2,$$

$$\tilde{g}^{s_1}_{S_0 H_1 \omega} = -\tilde{g}^{s_1}_{S_1 H_0 \omega} = -\tilde{g}^{s_1}_{S_1 H_1 \omega} = -0.36 \pm 0.03 \text{ GeV}^3,$$

$$\tilde{g}^{d_1}_{S_0 H_1 \omega} = -\tilde{g}^{d_1}_{S_1 H_0 \omega} = -\tilde{g}^{d_1}_{S_1 H_1 \omega} = -0.33 \pm 0.05 \text{ GeV},$$

$$\tilde{g}^{p_0}_{S_0 S_0 \omega} = -\tilde{g}^{p_0}_{S_1 S_1 \omega} = -0.29 \pm 0.03 \text{ GeV}^2,$$

$$\tilde{g}^{p_1}_{S_1 S_0 \omega} = -\tilde{g}^{p_1}_{S_1 S_1 \omega} = -0.38 \pm 0.02 \text{ GeV}^2,$$

$$\tilde{g}^{s_1}_{T_1 H_0 \omega} = -2\tilde{g}^{s_1}_{T_1 H_1 \omega} = -2\sqrt{\frac{2}{3}}\tilde{g}^{s_1}_{T_2 H_1 \omega} = -0.04 \pm 0.002 \text{ GeV}^4,$$

$$\tilde{g}^{d_1}_{T_1 H_0 \omega} = 2\tilde{g}^{d_1}_{T_1 H_1 \omega} = -2\sqrt{\frac{2}{3}}\tilde{g}^{d_1}_{T_2 H_1 \omega} = -0.43 \pm 0.05 \text{ GeV}^2,$$

$$\tilde{g}^{d_2}_{T_1 H_1 \omega} = \sqrt{\frac{3}{2}}\tilde{g}^{d_2}_{T_2 H_0 \omega} = \sqrt{6}\tilde{g}^{d_2}_{T_2 H_1 \omega} = -0.13 \pm 0.01 \text{ GeV}^2,$$

$$\tilde{g}^{p_1}_{T_1 S_0 \omega} = 2\tilde{g}^{p_1}_{T_1 S_1 \omega} = 2\sqrt{\frac{2}{3}}\tilde{g}^{p_1}_{T_2 S_1 \omega} = 0.15 \pm 0.01 \text{ GeV}^3,$$

$$\tilde{g}^{p_2}_{T_1 S_1 \omega} = -\sqrt{\frac{3}{2}}\tilde{g}^{p_2}_{T_2 S_0 \omega} = -\sqrt{6}\tilde{g}^{p_2}_{T_2 S_1 \omega} = -0.16 \pm 0.01 \text{ GeV}^3,$$

$$\tilde{g}^{f_2}_{T_1 S_1 \omega} = -\sqrt{\frac{3}{2}}\tilde{g}^{f_2}_{T_2 S_0 \omega} = -\sqrt{6}\tilde{g}^{f_2}_{T_2 S_1 \omega} = 0.17 \pm 0.02 \text{ GeV}, \tag{35}$$

$$g^{p_0}_{H_0 H_0 \omega} = -g^{p_0}_{H_1 H_1 \omega} = -4.6 \pm 0.6 \pm 1.2 \text{ GeV}^{-1},$$

$$g^{p_1}_{H_1 H_0 \omega} = g^{p_1}_{H_1 H_1 \omega} = -6.6 \pm 0.2 \pm 1.6 \text{ GeV}^{-1},$$

$$g^{s_1}_{S_0 H_1 \omega} = -g^{s_1}_{S_1 H_0 \omega} = -g^{s_1}_{S_1 H_1 \omega} = 3.6 \pm 0.3 \pm 0.9,$$

$$g^{d_1}_{S_0 H_1 \omega} = -g^{d_1}_{S_1 H_0 \omega} = -g^{d_1}_{S_1 H_1 \omega} = 3.3 \pm 0.5 \pm 0.8 \text{ GeV}^{-2},$$

$$g^{p_0}_{S_0 S_0 \omega} = -g^{p_0}_{S_1 S_1 \omega} = -1.8 \pm 0.2 \pm 0.5 \text{ GeV}^{-1},$$

$$g^{p_1}_{S_1 S_0 \omega} = -g^{p_1}_{S_1 S_1 \omega} = -2.4 \pm 0.1 \pm 0.6 \text{ GeV}^{-1},$$

$$g^{s_1}_{T_1 H_0 \omega} = -2g^{s_1}_{T_1 H_1 \omega} = -2\sqrt{\frac{2}{3}}g^{s_1}_{T_2 H_1 \omega} = -0.8 \pm 0.04 \pm 0.2,$$

$$g^{d_1}_{T_1 H_0 \omega} = 2g^{d_1}_{T_1 H_1 \omega} = -2\sqrt{\frac{2}{3}}g^{d_1}_{T_2 H_1 \omega} = -9.0 \pm 1.2 \pm 2.2 \text{ GeV}^{-2},$$

$$g^{d_2}_{T_1 H_1 \omega} = \sqrt{\frac{3}{2}}g^{d_2}_{T_2 H_0 \omega} = \sqrt{6}g^{d_2}_{T_2 H_1 \omega} = -2.8 \pm 0.1 \pm 0.7 \text{ GeV}^{-2},$$

$$g^{p_1}_{T_1 S_0 \omega} = 2g^{p_1}_{T_1 S_1 \omega} = 2\sqrt{\frac{2}{3}}g^{p_1}_{T_2 S_1 \omega} = 3.2 \pm 0.3 \pm 0.8 \text{ GeV}^{-1},$$

$$g^{p_2}_{T_1 S_1 \omega} = -\sqrt{\frac{3}{2}}g^{p_2}_{T_2 S_0 \omega} = -\sqrt{6}g^{p_2}_{T_2 S_1 \omega} = -3.3 \pm 0.3 \pm 0.8 \text{ GeV}^{-1},$$

$$g^{f_2}_{T_1 S_1 \omega} = -\sqrt{\frac{3}{2}} g^{f_2}_{T_2 S_0 \omega} = -\sqrt{6} g^{f_2}_{T_2 S_1 \omega} = 3.6 \pm 0.4 \pm 0.9 \text{ GeV}^{-3}. \tag{36}$$

IV. CONCLUSION

We have calculated the light vector meson couplings with heavy mesons in the leading order of HQET within the framework of LCQSR. The sum rules are stable with the variations of the Borel parameter and the continuum threshold. Some possible sources of the errors in our calculation include the inherent inaccuracy of LCQSR: the omission of the higher order terms in OPE, the choice of ω_c, the variation of the coupling constant with the Borel parameter T in the working interval and the approximation in the light-cone distribution amplitudes of the ρ meson. The uncertainty in f's and $\bar{\Lambda}$'s also leads to errors.

The extracted vector meson heavy meson coupling constants may be helpful in the study of the interaction between two $B(D)$ mesons. They may play an important role in the formation of these possible molecular candidates composed of two $B(D)$ mesons. They may also play a role in the interpretation of the proximity of X(3872), Y(4260) and Z(4430) to the threshold of two charmed mesons through the couple-channel mechanism.

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APPENDIX A: THE ρ DECAY AMPLITUDES OF HEAVY MESONS

The definitions of the ρ coupling constants not presented in the text are

$$\mathcal{M}(H_0 \to H_0 + \rho) = (e^* \cdot q_t) g^{p0}_{H_0 H_0 \rho}, \tag{A1}$$

$$\mathcal{M}(H_1 \to H_0 + \rho) = \epsilon^{\eta e^* q v} g^{p1}_{H_1 H_0 \rho}, \tag{A2}$$

$$\mathcal{M}(H_1 \to H_1 + \rho) = (e^* \cdot q_t)(\epsilon^* \cdot \eta_t)g^{p0}_{H_1 H_1 \rho} + \left[(e^* \cdot \eta_t)(\epsilon^* \cdot q_t) - (e^* \cdot \epsilon^*_t)(\eta \cdot q_t)\right]g^{p1}_{H_1 H_1 \rho}, \tag{A3}$$

$$\mathcal{M}(S_0 \to S_0 + \rho) = (e^* \cdot q_t)g^{p0}_{S_0 S_0 \rho}, \tag{A4}$$

$$\mathcal{M}(S_1 \to S_0 + \rho) = \epsilon^{\eta e^* q v}g^{p1}_{S_1 S_0 \rho}, \tag{A5}$$

$$\mathcal{M}(S_1 \to S_1 + \rho) = (e^* \cdot q_t)(\epsilon^* \cdot \eta_t)g^{p0}_{S_1 S_1 \rho} + \left[(e^* \cdot \eta_t)(\epsilon^* \cdot q_t) - (e^* \cdot \epsilon^*_t)(\eta \cdot q_t)\right]g^{p1}_{S_1 S_1 \rho}, \tag{A6}$$

$$\mathcal{M}(S_0 \to H_1 + \rho) = (e^* \cdot \epsilon^*_t)g^{s1}_{S_0 H_1 \rho} + \left[(\epsilon^* \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(e^* \cdot \epsilon^*_t)q^2_t\right]g^{d1}_{S_0 H_1 \rho}, \tag{A7}$$

$$\mathcal{M}(S_1 \to H_0 + \rho) = (e^* \cdot \eta_t)g^{s1}_{S_1 H_0 \rho} + \left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(e^* \cdot \eta_t)q^2_t\right]g^{d1}_{S_1 H_0 \rho}, \tag{A8}$$

$$\mathcal{M}(S_1 \to H_1 + \rho) = \epsilon^{\eta \epsilon^* e^* v}g^{s1}_{S_1 H_1 \rho} + \left[\epsilon^{\eta \epsilon^* q v}(e^* \cdot q_t) - \frac{1}{3}\epsilon^{\eta \epsilon^* e^* v}q^2_t\right]g^{d1}_{S_1 H_1 \rho}, \tag{A9}$$

$$\mathcal{M}(T_1 \to H_0 + \rho) = (e^* \cdot \eta_t)g^{s1}_{T_1 H_0 \rho} + \left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(e^* \cdot \eta_t)q^2_t\right]g^{d1}_{T_1 H_0 \rho}, \tag{A10}$$

$$\mathcal{M}(T_1 \to H_1 + \rho) = \epsilon^{\eta \epsilon^* e^* v}g^{s1}_{T_1 H_1 \rho} + \left[\epsilon^{\eta \epsilon^* q v}(e^* \cdot q_t) - \frac{1}{3}\epsilon^{\eta \epsilon^* e^* v}q^2_t\right]g^{d1}_{T_1 H_1 \rho}$$
$$+ \left[\epsilon^{\eta e q v}(q_t \cdot \epsilon^*) + \epsilon^{\epsilon^* e q v}(q_t \cdot \eta)\right]g^{d2}_{T_1 H_1 \rho}, \tag{A11}$$

$$\mathcal{M}(T_1 \to S_0 + \rho) = \epsilon^{\eta e^* q v}g^{p1}_{T_1 S_0 \rho}, \tag{A12}$$

$$\mathcal{M}(T_1 \to S_1 + \rho) = \left[(e^* \cdot \eta_t)(\epsilon^* \cdot q_t) - (e^* \cdot \epsilon^*_t)(\eta \cdot q_t)\right]g^{p1}_{T_1 S_1 \rho}$$
$$+ \left[(e^*_t \cdot \eta)(q_t \cdot \epsilon^*) + (q_t \cdot \eta)(e^*_t \cdot \epsilon^*) - \frac{2}{3}(e^*_t \cdot \eta)(e^* \cdot q_t)\right]g^{p2}_{T_1 S_1 \rho}$$
$$+ \Big\{ (q_t \cdot \eta)(q_t \cdot \epsilon^*)(e^* \cdot q_t)$$
$$- \frac{q^2_t}{5}\left[(\eta_t \cdot \epsilon^*)(e^* \cdot q_t) + (e^*_t \cdot \eta)(q_t \cdot \epsilon^*) + (q_t \cdot \eta)(e^*_t \cdot \epsilon^*)\right] \Big\}g^{f2}_{T_1 S_1 \rho}, \tag{A13}$$

$$\mathcal{M}(T_2 \to H_0 + \rho) = \eta_{\alpha_1 \alpha_2}(\epsilon^{\alpha_1 e^* q v}q^{\alpha_2}_t + \epsilon^{\alpha_2 e^* q v}q^{\alpha_1}_t)g^{d2}_{T_2 H_0 \rho}, \tag{A14}$$

$$\mathcal{M}(T_2 \to H_1 + \rho) = \eta_{\alpha_1 \alpha_2}\left[\epsilon^{*\alpha_1}e^{*\alpha_2}_t + \epsilon^{*\alpha_2}e^{*\alpha_1}_t - \frac{2}{3}g^{\alpha_1 \alpha_2}_t(e^*_t \cdot \epsilon^*)\right]g^{s1}_{T_2 H_1 \rho}$$
$$+ \eta_{\alpha_1 \alpha_2}\Big\{ \left[\epsilon^{*\alpha_1}q^{\alpha_2}_t + \epsilon^{*\alpha_2}q^{\alpha_1}_t - \frac{2}{3}g^{\alpha_1 \alpha_2}_t(q_t \cdot \epsilon^*)\right](e^* \cdot q_t)$$
$$- \frac{1}{3}\left[\epsilon^{*\alpha_1}e^{*\alpha_2}_t + \epsilon^{*\alpha_2}e^{*\alpha_1}_t - \frac{2}{3}g^{\alpha_1 \alpha_2}_t(e^*_t \cdot \epsilon^*)\right]q^2_t \Big\}g^{d1}_{T_2 H_1 \rho}$$
$$+ \eta_{\alpha_1 \alpha_2}\Big\{ 2\left[e^{*\alpha_1}_t q^{\alpha_2}_t(q_t \cdot \epsilon^*) + q^{\alpha_1}_t e^{*\alpha_2}_t(q_t \cdot \epsilon^*) - 2q^{\alpha_1}_t q^{\alpha_2}_t(e^*_t \cdot \epsilon^*)\right]$$
$$+ \left[\epsilon^{*\alpha_1}_t q^{\alpha_2}_t + \epsilon^{*\alpha_2}_t q^{\alpha_1}_t - 2g^{\alpha_1 \alpha_2}_t(q_t \cdot \epsilon^*)\right](e^* \cdot q_t)$$
$$- \left[\epsilon^{*\alpha_1}_t e^{*\alpha_2}_t + \epsilon^{*\alpha_2}_t e^{*\alpha_1}_t - 2g^{\alpha_1 \alpha_2}_t(e^*_t \cdot \epsilon^*)\right]q^2_t \Big\}g^{d2}_{T_2 H_1 \rho}, \tag{A15}$$

$$\mathcal{M}(T_2 \to S_0 + \rho) = \eta_{\alpha_1 \alpha_2}\left[e^{*\alpha_1}_t q^{\alpha_2}_t + q^{\alpha_1}_t e^{*\alpha_2}_t - \frac{2}{3}g^{\alpha_1 \alpha_2}_t(e^* \cdot q_t)\right]g^{p2}_{T_2 S_0 \rho}$$
$$+ \eta_{\alpha_1 \alpha_2}\Big\{ q^{\alpha_1}_t q^{\alpha_2}_t(e^* \cdot q_t) - \frac{q^2_t}{5}\left[g^{\alpha_1 \alpha_2}_t(e^* \cdot q_t) + e^{*\alpha_1}_t q^{\alpha_2}_t + q^{\alpha_1}_t e^{*\alpha_2}_t\right] \Big\}g^{f2}_{T_2 S_0 \rho}, \tag{A16}$$

$$\mathcal{M}(T_2 \to S_1 + \rho) = \eta_{\alpha_1 \alpha_2}\left[-\epsilon^{\alpha_1 e^* q v}\epsilon^{*\alpha_2}_t - \epsilon^{\alpha_2 e^* q v}\epsilon^{*\alpha_1}_t + \frac{2}{3}g^{\alpha_1 \alpha_2}_t\epsilon^{\epsilon^* e^* q v}\right]g^{p1}_{T_2 S_1 \rho}$$
$$+ \eta_{\alpha_1 \alpha_2}\left[\epsilon^{\alpha_1 \epsilon^* e^* v}q^{\alpha_2}_t + \epsilon^{\alpha_2 \epsilon^* e^* v}q^{\alpha_1}_t + \epsilon^{\alpha_1 \epsilon^* q v}e^{\alpha_2}_t + \epsilon^{\alpha_2 \epsilon^* q v}e^{*\alpha_1}_t\right]g^{p2}_{T_2 S_1 \rho}$$
$$+ \eta_{\alpha_1 \alpha_2}\Big\{ \epsilon^{\alpha_1 \epsilon^* q v}q^{\alpha_2}_t(e^* \cdot q_t) + \epsilon^{\alpha_2 \epsilon^* q v}q^{\alpha_1}_t(e^* \cdot q_t)$$
$$- \frac{q^2_t}{5}\left[\epsilon^{\alpha_1 \epsilon^* q v}e^{*\alpha_2}_t + \epsilon^{\alpha_2 \epsilon^* q v}e^{*\alpha_1}_t + \epsilon^{\alpha_1 \epsilon^* e^* v}q^{\alpha_2}_t + \epsilon^{\alpha_2 \epsilon^* e^* v}q^{\alpha_1}_t\right] \Big\}g^{f2}_{T_2 S_1 \rho}. \tag{A17}$$

Note that these decay amplitudes may be organized in another way. For example, the tensor structure corresponding to $g^{p0}_{H_1 H_1 \rho}$ was defined as $(e^* \cdot v)(\epsilon^* \cdot \eta_t)$ in equation (28) of Ref. [8] rather than $(e^* \cdot q_t)(\epsilon^* \cdot \eta_t)$ in Eq. (A3). Since we have $(e^* \cdot q_t) = -(q \cdot v)(e^* \cdot v)$, the essentially same sum rule as Eq. (24) of Ref. [8] can be obtained if we isolate the

tensor structure $(e^* \cdot v)(\epsilon^* \cdot \eta_t)$.

To derive sum rules for these coupling constants, we consider the following correlators:

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,-,\frac{1}{2}}(0)J^\dagger_{0,-,\frac{1}{2}}(x)\}|0\rangle = (e^* \cdot q_t)G^{p0}_{H_0 H_0 \rho}(\omega,\omega'), \tag{A18}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,-,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,-,\frac{1}{2}}(x)\}|0\rangle = \epsilon^{\alpha e^* qv}G^{p1}_{H_1 H_0 \rho}(\omega,\omega'), \tag{A19}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J^\beta_{1,-,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,-,\frac{1}{2}}(x)\}|0\rangle = g_t^{\alpha\beta}(e^* \cdot q_t)G^{p0}_{H_1 H_1 \rho}(\omega,\omega')$$
$$+(e_t^{*\alpha}q_t^\beta - q_t^\alpha e_t^{*\beta})G^{p1}_{H_1 H_1 \rho}(\omega,\omega'), \tag{A20}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,+,\frac{1}{2}}(0)J^\dagger_{0,+,\frac{1}{2}}(x)\}|0\rangle = (e^* \cdot q_t)G^{p0}_{S_0 S_0 \rho}(\omega,\omega'), \tag{A21}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,+,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,+,\frac{1}{2}}(x)\}|0\rangle = \epsilon^{\alpha e^* qv}G^{p1}_{S_1 S_0 \rho}(\omega,\omega'), \tag{A22}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J^\beta_{1,+,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,+,\frac{1}{2}}(x)\}|0\rangle = g_t^{\alpha\beta}(e^* \cdot q_t)G^{p0}_{S_1 S_1 \rho}(\omega,\omega') + (e_t^{*\alpha}q_t^\beta - q_t^\alpha e_t^{*\beta})G^{p1}_{S_1 S_1 \rho}(\omega,\omega'), \tag{A23}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J^\beta_{1,-,\frac{1}{2}}(0)J^\dagger_{0,+,\frac{1}{2}}(x)\}|0\rangle = e_t^{*\beta}G^{s1}_{S_0 H_1 \rho}(\omega,\omega') + \left[q_t^\beta(e^* \cdot q_t) - \frac{1}{3}e_t^{*\beta}q_t^2\right]G^{d1}_{S_0 H_1 \rho}(\omega,\omega'), \tag{A24}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,-,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,+,\frac{1}{2}}(x)\}|0\rangle = e_t^{*\alpha}G^{s1}_{S_1 H_0 \rho}(\omega,\omega') + \left[q_t^\alpha(e^* \cdot q_t) - \frac{1}{3}e_t^{*\alpha}q_t^2\right]G^{d1}_{S_1 H_0 \rho}(\omega,\omega'), \tag{A25}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J^\beta_{1,-,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,+,\frac{1}{2}}(x)\}|0\rangle = \epsilon^{\alpha\beta e^* v}G^{s1}_{S_1 H_1 \rho}(\omega,\omega')$$
$$+\left[\epsilon^{\alpha\beta qv}(e^* \cdot q_t) - \frac{1}{3}\epsilon^{\alpha\beta e^* v}q_t^2\right]G^{d1}_{S_1 H_1 \rho}(\omega,\omega'), \tag{A26}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,-,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,+,\frac{3}{2}}(x)\}|0\rangle = e_t^{*\alpha}G^{s1}_{T_1 H_0 \rho}(\omega,\omega') + \left[q_t^\alpha(e^* \cdot q_t) - \frac{1}{3}e_t^{*\alpha}q_t^2\right]G^{d1}_{T_1 H_0 \rho}(\omega,\omega'), \tag{A27}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,+,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,+,\frac{3}{2}}(x)\}|0\rangle = \epsilon^{\alpha e^* qv}G^{p1}_{T_1 S_0 \rho}(\omega,\omega'), \tag{A28}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J^\beta_{1,+,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,+,\frac{3}{2}}(x)\}|0\rangle = g_t^{\alpha\beta}(e^* \cdot q_t)G^{p0}_{T_1 S_1 \rho}(\omega,\omega') + (e_t^{*\alpha}q_t^\beta - q_t^\alpha e_t^{*\beta})G^{p1}_{T_1 S_1 \rho}(\omega,\omega'), \tag{A29}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,-,\frac{1}{2}}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,\frac{3}{2}}(x)\}|0\rangle = (\epsilon^{\alpha_1 e^* qv}q_t^{\alpha_2} + \epsilon^{\alpha_2 e^* qv}q_t^{\alpha_1})G^{d2}_{T_2 H_0 \rho}(\omega,\omega'), \tag{A30}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J^\beta_{1,-,\frac{1}{2}}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,\frac{3}{2}}(x)\}|0\rangle$$
$$= \left[g_t^{\alpha_1\beta}e_t^{*\alpha_2} + g_t^{\alpha_2\beta}e_t^{*\alpha_1} - \frac{2}{3}g_t^{\alpha_1\alpha_2}e_t^{*\beta}\right]G^{s1}_{T_2 H_1 \rho}(\omega,\omega')$$
$$+\left\{\left[g_t^{\alpha_1\beta}q_t^{\alpha_2} + g_t^{\alpha_2\beta}q_t^{\alpha_1} - \frac{2}{3}g_t^{\alpha_1\alpha_2}q_t^\beta\right](e^* \cdot q_t) - \frac{1}{3}\left[g_t^{\alpha_1\beta}e_t^{*\alpha_2} + g_t^{\alpha_2\beta}e_t^{*\alpha_1} - \frac{2}{3}g_t^{\alpha_1\alpha_2}e_t^{*\beta}\right]q_t^2\right\}G^{d1}_{T_2 H_1 \rho}(\omega,\omega')$$
$$+\left[2(e_t^{*\alpha_1}q_t^{\alpha_2}q_t^\beta + q_t^{\alpha_1}e_t^{*\alpha_2}q_t^\beta - 2q_t^{\alpha_1}q_t^{\alpha_2}e_t^{*\beta}) + (g_t^{\alpha_1\beta}q_t^{\alpha_2} + g_t^{\alpha_2\beta}q_t^{\alpha_1} - 2g_t^{\alpha_1\alpha_2}q_t^\beta)(e^* \cdot q_t)\right.$$
$$\left.-(g_t^{\alpha_1\beta}e_t^{*\alpha_2} + g_t^{\alpha_2\beta}e_t^{*\alpha_1} - 2g_t^{\alpha_1\alpha_2}e_t^{*\beta})q_t^2\right]G^{d2}_{T_2 H_1 \rho}(\omega,\omega'), \tag{A31}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J_{0,+,\frac{1}{2}}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,\frac{3}{2}}(x)\}|0\rangle$$
$$= \left[e_t^{*\alpha_1}q_t^{\alpha_2} + q_t^{\alpha_1}e_t^{*\alpha_2} - \frac{2}{3}g_t^{\alpha_1\alpha_2}(e^* \cdot q_t)\right]G^{p2}_{T_2 S_0 \rho}(\omega,\omega')$$
$$+\left\{q_t^{\alpha_1}q_t^{\alpha_2}(e^* \cdot q_t) - \frac{q_t^2}{5}\left[g_t^{\alpha_1\alpha_2}(e^* \cdot q_t) + e_t^{*\alpha_1}q_t^{\alpha_2} + q_t^{\alpha_1}e_t^{*\alpha_2}\right]\right\}G^{f2}_{T_2 S_0 \rho}(\omega,\omega'), \tag{A32}$$

$$\int d^4x e^{-ik\cdot x} \langle \rho(q)|T\{J^\beta_{1,+,\frac{1}{2}}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,\frac{3}{2}}(x)\}|0\rangle$$
$$= \left[-\epsilon^{\alpha_1 e^* qv}g_t^{\alpha_2\beta} - \epsilon^{\alpha_2 e^* qv}g_t^{\alpha_1\beta} + \frac{2}{3}g_t^{\alpha_1\alpha_2}\epsilon^{\beta e^* qv}\right]G^{p1}_{T_2 S_1 \rho}(\omega,\omega')$$

$$+\left[\epsilon^{\alpha_1\beta e^* v}q_t^{\alpha_2}+\epsilon^{\alpha_2\beta e^* v}q_t^{\alpha_1}+\epsilon^{\alpha_1\beta qv}e_t^{*\alpha_2}+\epsilon^{\alpha_2\beta qv}e_t^{*\alpha_1}\right]G^{p2}_{T_2S_1\rho}(\omega,\omega')$$

$$+\left\{\epsilon^{\alpha_1\beta qv}q_t^{\alpha_2}(e^*\cdot q_t)+\epsilon^{\alpha_2\beta qv}q_t^{\alpha_1}(e^*\cdot q_t)\right.$$

$$\left.-\frac{q_t^2}{5}\left[\epsilon^{\alpha_1\beta qv}e_t^{*\alpha_2}+\epsilon^{\alpha_2\beta qv}e_t^{*\alpha_1}+\epsilon^{\alpha_1\beta e^* v}q_t^{\alpha_2}+\epsilon^{\alpha_2\beta e^* v}q_t^{\alpha_1}\right]\right\}G'^{f2}_{T_2S_1\rho}(\omega,\omega') \tag{A33}$$

APPENDIX B: THE ρ MESON LIGHT-CONE DISTRIBUTION AMPLITUDES

The definitions of the distribution amplitudes used in the text read as [13]

$$\langle 0|\bar{u}(z)\gamma_\mu d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho\left[p_\mu\frac{e^{(\lambda)}\cdot z}{p\cdot z}\int_0^1 du\,e^{i\xi p\cdot z}\phi_\|(u,\mu^2)+e^{(\lambda)}_{\perp\mu}\int_0^1 du\,e^{i\xi p\cdot z}g^{(v)}_\perp(u,\mu^2)\right.$$

$$\left.-\frac{1}{2}z_\mu\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_\rho^2\int_0^1 du\,e^{i\xi p\cdot z}g_3(u,\mu^2)\right], \tag{B1}$$

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = \frac{1}{2}f_\rho m_\rho\epsilon_\mu^{\ \nu\alpha\beta}e^{(\lambda)}_{\perp\nu}p_\alpha z_\beta\int_0^1 du\,e^{i\xi p\cdot z}g^{(a)}_\perp(u,\mu^2), \tag{B2}$$

$$\langle 0|\bar{u}(z)\sigma_{\mu\nu}d(-z)|\rho^-(P,\lambda)\rangle = if_\rho^T\left[(e^{(\lambda)}_{\perp\mu}p_\nu-e^{(\lambda)}_{\perp\nu}p_\mu)\int_0^1 du\,e^{i\xi p\cdot z}\phi_\perp(u,\mu^2)\right.$$

$$+(p_\mu z_\nu-p_\nu z_\mu)\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_\rho^2\int_0^1 du\,e^{i\xi p\cdot z}h^{(t)}_\|(u,\mu^2)$$

$$\left.+\frac{1}{2}(e^{(\lambda)}_{\perp\mu}z_\nu-e^{(\lambda)}_{\perp\nu}z_\mu)\frac{m_\rho^2}{p\cdot z}\int_0^1 du\,e^{i\xi p\cdot z}h_3(u,\mu^2)\right], \tag{B3}$$

$$\langle 0|\bar{u}(z)d(-z)|\rho^-(P,\lambda)\rangle = -if_\rho^T(e^{(\lambda)}z)m_\rho^2\int_0^1 du\,e^{i\xi p\cdot z}h^{(s)}_\|(u,\mu^2). \tag{B4}$$

The vector and tensor decay constants f_ρ and f_ρ^T are defined as

$$\langle 0|\bar{u}(0)\gamma_\mu d(0)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho e^{(\lambda)}_\mu, \tag{B5}$$

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu}d(0)|\rho^-(P,\lambda)\rangle = if_\rho^T(e^{(\lambda)}_\mu P_\nu-e^{(\lambda)}_\nu P_\mu). \tag{B6}$$

The distribution amplitude $\phi_\|$ and $\phi_\perp$ are of twist-2, $g^{(v)}_\perp$, $g^{(a)}_\perp$, $h^{(s)}_\|$ and $h^{(t)}_\|$ are twist-3 and g_3, h_3 are twist-4. All functions $\phi=\{\phi_\|,\phi_\perp,g^{(v)}_\perp,g^{(a)}_\perp,h^{(s)}_\|,h^{(t)}_\|,g_3,h_3\}$ are normalized to satisfy $\int_0^1 du\,\phi(u)=1$.

The 3-particle distribution amplitudes are defined as [13]

$$\langle 0|\bar{u}(z)g\widetilde{G}_{\mu\nu}\gamma_\alpha\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu}-p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{A}(v,pz)$$

$$+f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu}-p_\nu g^\perp_{\alpha\mu}]\widetilde{\Phi}(v,pz)$$

$$+f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{(pz)^2}p_\alpha[p_\mu z_\nu-p_\nu z_\mu]\widetilde{\Psi}(v,pz) \tag{B7}$$

$$\langle 0|\bar{u}(z)gG_{\mu\nu}i\gamma_\alpha d(-z)|\rho^-(P)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu}-p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{V}(v,pz)$$

$$+f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu}-p_\nu g^\perp_{\alpha\mu}]\Phi(v,pz)$$

$$+f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{(pz)^2}p_\alpha[p_\mu z_\nu-p_\nu z_\mu]\Psi(v,pz), \tag{B8}$$

$$\langle 0|\bar{u}(z)\sigma_{\alpha\beta}gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle$$

$$
\begin{aligned}
= \; & f_\rho^T m_\rho^3 \frac{e^{(\lambda)} \cdot z}{2(p \cdot z)} [p_\alpha p_\mu g_{\beta\nu}^\perp - p_\beta p_\mu g_{\alpha\nu}^\perp - p_\alpha p_\nu g_{\beta\mu}^\perp + p_\beta p_\nu g_{\alpha\mu}^\perp] \mathcal{T}(v, pz) \\
& + f_\rho^T m_\rho^2 [p_\alpha e_{\perp\mu}^{(\lambda)} g_{\beta\nu}^\perp - p_\beta e_{\perp\mu}^{(\lambda)} g_{\alpha\nu}^\perp - p_\alpha e_{\perp\nu}^{(\lambda)} g_{\beta\mu}^\perp + p_\beta e_{\perp\nu}^{(\lambda)} g_{\alpha\mu}^\perp] T_1^{(4)}(v, pz) \\
& + f_\rho^T m_\rho^2 [p_\mu e_{\perp\alpha}^{(\lambda)} g_{\beta\nu}^\perp - p_\mu e_{\perp\beta}^{(\lambda)} g_{\alpha\nu}^\perp - p_\nu e_{\perp\alpha}^{(\lambda)} g_{\beta\mu}^\perp + p_\nu e_{\perp\beta}^{(\lambda)} g_{\alpha\mu}^\perp] T_2^{(4)}(v, pz) \\
& + \frac{f_\rho^T m_\rho^2}{pz} [p_\alpha p_\mu e_{\perp\beta}^{(\lambda)} z_\nu - p_\beta p_\mu e_{\perp\alpha}^{(\lambda)} z_\nu - p_\alpha p_\nu e_{\perp\beta}^{(\lambda)} z_\mu + p_\beta p_\nu e_{\perp\alpha}^{(\lambda)} z_\mu] T_3^{(4)}(v, pz) \\
& + \frac{f_\rho^T m_\rho^2}{pz} [p_\alpha p_\mu e_{\perp\nu}^{(\lambda)} z_\beta - p_\beta p_\mu e_{\perp\nu}^{(\lambda)} z_\alpha - p_\alpha p_\nu e_{\perp\mu}^{(\lambda)} z_\beta + p_\beta p_\nu e_{\perp\mu}^{(\lambda)} z_\alpha] T_4^{(4)}(v, pz) \\
& + \dots
\end{aligned}
\tag{B9}
$$

$$
\begin{aligned}
\langle 0 | \bar{u}(z) g G_{\mu\nu}(vz) d(-z) | \rho^-(P, \lambda) \rangle &= i f_\rho^T m_\rho^2 [e_{\perp\mu}^{(\lambda)} p_\nu - e_{\perp\nu}^{(\lambda)} p_\mu] S(v, pz), \\
\langle 0 | \bar{u}(z) i g \widetilde{G}_{\mu\nu}(vz) \gamma_5 d(-z) | \rho^-(P, \lambda) \rangle &= i f_\rho^T m_\rho^2 [e_{\perp\mu}^{(\lambda)} p_\nu - e_{\perp\nu}^{(\lambda)} p_\mu] \widetilde{S}(v, pz).
\end{aligned}
\tag{B10}
$$

where

$$
\mathcal{A}(v, pz) = \int \mathcal{D}\underline{\alpha} \, e^{-ipz(\alpha_u - \alpha_d + v\alpha_g)} \mathcal{A}(\underline{\alpha}),
\tag{B11}
$$

etc. The integration measure is

$$
\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_d \int_0^1 d\alpha_u \int_0^1 d\alpha_g \delta\left(1 - \sum \alpha_i\right)
\tag{B12}
$$

The distribution amplitudes $\mathcal{A}$, $\mathcal{V}$ and $\mathcal{T}$ are twist-3 and the others are twist-4.